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KOVALEVSKAYA'S METHOD IN RIGID BODY DYNAMICS†

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An example from the field of rigid body dynamics, possessing a natural physical justification, is presented. The behaviour of the solutions of the equations of motion in the real domain, whatever the initial data, is regular; nevertheless, depending on the values of a certain control parameter, the solution of the system may branch in the complex time plane, and the system will have multi-valued first integrals. A denumerable sequence of single-valued polynomial integrals of arbitrarily high even degree is found (unlike Kovalevskaya's case, in which the degree of the first integral of the Euler–Poisson equations is four). As an extension, a system from non-holonomic mechanics is considered.

It is well known that the equations of motion in the classical problems of rigid body dynamics (the Euler–Poisson equations, Kirchhoff's equations, and the Poincaré–Lamb–Zhukovskii equations) [1, 2] may be expressed in quasi-homogeneous form (in the senses of the definition in [3]). Since Kovalevskaya's time, to seek integrable cases of such systems a method has been used based on studying the branching of the general solution in the complex time plane. When the general solution was meromorphic, this was associated with the existence of an additional algebraic first integral and with the integrability of the system using theta-functions. Development of this idea led to Husson's method of proving that no additional algebraic integral exists and to the methods described in [4–6], in which obstacles to the existence of an additional first integral, single-valued in the complex plane, were found. These results convinced specialists in rigid body dynamics that the integrability of the equations of motion, and hence also their regular behaviour, were due to the existence of single-valued integrals and their integrability in theta-functions.

1. QUASI-HOMOGENEOUS SYSTEMS AND KOVALEVSKAYA INDICES

We shall call a system of n differential equations

$$\dot{x}_i = v_i(x_1, \ldots, x_n), \quad i = 1, \ldots, n \tag{1.1}$$

quasi-homogeneous with quasi-homogeneity indices $g_1, \ldots, g_n \in Q$ if

$$v_i(\alpha^{g_1}x_1, \ldots, \alpha^{g_n}x_n) = \alpha^{g_i+1}v_i(x_1, \ldots, x_n) \tag{1.2}$$

for all values of x, and $\alpha > 0$. For a quasi-homogeneous first integral $\Phi(x_1, \ldots, x_n)$ of system (1.1), it must be true that

$$\Phi(\alpha^{g_1}x_1, \ldots, \alpha^{g_n}x_n) = \alpha^g\Phi(x_1, \ldots, x_n) \tag{1.3}$$

where the number $g \in \mathbb{R}$ is known as the quasi-homogeneity index.

As an example, we can consider system (1.1) with homogeneous quadratic right-hand sides. In that case $g_1 = g_n = 1$. In rigid body dynamics, Kirchhoff's equations, describing the motion of a simply-connected rigid body in an unlimited volume of an irrational ideal incompressible fluid [12], can be reduced to this class of systems. The same is true of the Poincaré–Lamb–Zhukovskii equations governing the motion about a fixed point of a rigid body having ellipsoid cavities filled with a homogeneous turbulent ideal fluid [2].

A modification of Kovalevskaya's method has been applied to quasi-homogeneous systems, and the so-called Kovalevskaya indices, characterizing the expansion of the general solution near a singular point

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of a system of differential equations (if the general solution is meromorphic, these indices are integers), were associated with the degrees of quasi-homogeneity of the first integrals [3]. For further application, we will reformulate the results previously obtained in [3, 7] more rigorously.

Equations (1.1) have a particular solution of the form

$$x_i = c_1 t^{-g_1}, \ldots, x_n = c_n t^{-g_n},$$

where the constant coefficients c_i (complex in the general case) satisfy an algebraic system of equations

$$v_i(c_1, \ldots, c_n) = -g_i c_i, \quad i = 1, 2, \ldots, n \tag{1.4}$$

In real systems, one can find certain particular non-trivial solutions of (1.4). The Kovalevskaya indices (KIs) are defined as the eigenvalues $\rho_1, \ldots, \rho_n$ of the matrix

$$\left\| \frac{\partial \mathbf{v}}{\partial \mathbf{x}}(\mathbf{c}) + g \right\|; \quad \mathbf{c} = (c_1, \ldots, c_n), \quad g = \operatorname{diag}(g_1, \ldots, g_n). \tag{1.5}$$

Since system (1.1) is autonomous, one of the KIs is always equal to –1. As is easily shown, the sum of all KIs for a given vector **c** is constant and equal to the dimensionality of the system. The relationship between the degrees of quasi-homogeneity of first integrals, fields of symmetry and KIs is given by the following theorems.

Theorem 1 [3]. If Φ is a quasi-homogeneous integral of system (1.1) and grad $\Phi(\mathbf{c})$ is neither zero nor infinity, the quasi-homogeneous degree of the integral ρ is a KI of the system.

Theorem 2 [7]. If $[\mathbf{u}, \mathbf{v}] = 0$ (i.e. **u** is a field of symmetries of system (1.1)) and **u** is quasi-homogeneous, deg (u_i/x_i) are equal to one another, $i = 1, \ldots, n$, and $\mathbf{u}(\mathbf{c}) \neq 0$, then –deg (u_i/x_i) is a KI of the system. If there are several fields of integrals (or fields of symmetries) and they are independent of the solution (1.4), the quasi-homogeneous degree of each of them is a KI with the appropriate multiplicity.

The investigation of the existence of quasi-homogeneous integrals for quasi-homogeneous systems (1.1) is in fact of more general interest, because any first integral of such a system that is meromorphic in $\mathbb{C}^n$ reduces to a rational quasi-homogeneous integral if the signs of the g_i are the same, or to a polynomial quasi-homogeneous integral if it has no singular points. Note that a quasi-homogeneous integral may have an irrational or complex degree of quasi-homogeneity, in which case it will not be single-valued. In the following sections we will present examples in which multi-valued integrals exist.

2. EULER'S EQUATIONS ON SO(4) AND THE POINCARÉ–LAMB–ZHUKOVSKII EQUATIONS

It is well known that Euler's equations, describing the free rotation of a four-dimensional rigid body about a fixed point, are identical with the classical Poincaré–Lamb–Zhukovskii equations [8] (more precisely, they include them). The equations of this problem may be considered as approximate equations for the motion of the Earth, which has a solid mantle and a liquid core (for details, see [9]). The equations of motion of a top on $SO(4)$ have also been analysed in the physics literature [10, 11], where the dynamics of interacting spins in an external field have been considered; these publications partly overlap results of research carried out in an even more general situation (see, for example [12]).

The Lie algebra $SO(4)$ is not simple and may be expressed as a direct sum $SO(3) \oplus SO(3)$. One can imagine this as meaning that one copy of $SO(3)$ corresponds to the rotation of a solid about a fixed point, while the other corresponds to the quasi-rigid motion of a fluid (in Helmholtz's sense).

If M_i and γ_i are the components of the angular momentum and vorticity vectors, respectively, in a system of coordinates attached to the body, then the equations of motion of the system may be written as

$$\begin{aligned} \mathbf{M} &= \mathbf{M} \times \partial H / \partial \mathbf{M} = \mathbf{M} \times (A\mathbf{M} + B\boldsymbol{\gamma}) \\ \dot{\boldsymbol{\gamma}} &= \boldsymbol{\gamma} \times \partial H / \partial \boldsymbol{\gamma} = \boldsymbol{\gamma} \times (B\mathbf{M} + C\boldsymbol{\gamma}), \end{aligned} \tag{2.1}$$

where the Hamiltonian H is a homogeneous quadratic form in the variables $\mathbf{M}$, $\boldsymbol{\gamma}$

$$H=(A\mathbf{M},\mathbf{M})/2+(B\mathbf{M},\boldsymbol{\gamma})+(C\boldsymbol{\gamma},\boldsymbol{\gamma})/2. \tag{2.2}$$

The coefficients of the matrices A, B and C are determined by the dynamical characteristics of the body and the geometrical characteristics of the ellipsoidal cavity. Henceforth we shall assume that they are diagonal.

Note that Eqs (2.1) may be expressed as Hamilton equations

$$\dot{\mathbf{x}}=\{\mathbf{x},H\},\quad \mathbf{x}=(\mathbf{M},\boldsymbol{\gamma})\in\mathbb{R}^6 \tag{2.3}$$

with a (degenerate) Poisson bracket defined by the commutation relations of the algebra $SO(4)\approx SO(3)\oplus SO(3)$

$$\{M_i,M_j\}=\varepsilon_{ijk}M_k,\quad \{\gamma_i,\gamma_i\}=\varepsilon_{ijk}\gamma_k,\quad \{M_i,\gamma_j\}=0 \tag{2.4}$$

where ε_{ijk} is the Levi–Civita symbol.

Equations (2.1) always have, besides the energy integral $H=E=$ const, another two first integrals (annihilators of the Poisson structure) $(\mathbf{M},\mathbf{M})=M_0^2$, $(\gamma,\gamma)=\gamma_0^2$, restricted to which the Poisson structure (2.4) becomes non-degenerate. On the surfaces $\mathbf{M}^2=M_0^2$, $\gamma^2=\gamma_0^2$, system (2.1) reduces to the usual autonomous Hamiltonian system with two degrees of freedom, and a necessary condition for its integrability is that one more first integral exists.

Let us evaluate the KIs for the Eqs (2.1). The following two vectors satisfy system (1.4) for constant c_i

$$\begin{gathered}(\pm(a_{13}a_{21})^{-1/2},\ \mp(a_{21}a_{32})^{-1/2},\ \pm(a_{32}a_{13})^{-1/2},0,0,0)\\ (0,0,0,\pm(c_{13}c_{21})^{-1/2},\ \mp(c_{21}c_{32})^{-1/2},\ \pm(c_{32}c_{13})^{-1/2}),\end{gathered} \tag{2.5}$$

where $a_{ij}=a_i-a_j$, $c_{ij}=c_i-c_j$. These solutions exist provided that all the elements of the matrices A and C are distinct. For the KIs of the two different solutions (2.5) we have

$$\begin{gathered}\rho_1=-1,\quad \rho_2=\rho_3=2,\quad \rho_4=1,\quad \rho_{5,6}=1\pm k_d\\ k_d=\left[-\frac{b_1d_{32}^2+b_2d_{13}^2+b_3d_{21}^2}{d_{21}d_{32}d_{13}}\right]^{-1/2},\quad d=a,c.\end{gathered} \tag{2.6}$$

A necessary condition for all these KIs to be integers is that

$$b_1^2d_{32}+b_2^2d_{13}+b_3^2d_{21}+k_1^2d_{21}d_{32}d_{13}=0,\quad d=a,c, \tag{2.7}$$

where $k_d\in\mathbb{N}$; these conditions are identical with those in [7] (which were derived from the conditions that the general solution be meromorphic in the complex time plane). According to Theorem 1, the quantities $\rho=1+k_a$ and $\rho=1+k_c$ are candidates for the role of "good" degrees (for which grad $\Phi(c)\neq 0$) of first "good" integrals, if the latter exist. Quadratic integrals of system (2.1) were discovered and generalized in publications of Fram, Schottky, Steklov and Manakov (see [12, 13]).

A general integrable case, with $k_a=3$, $k_c=1$ was mentioned in [14]. There the additional integral is of fourth degree, and additional conditions (besides (2.7)) are imposed on the elements of the matrices A, B and C.

3. RESTRICTED FORMULATION OF THE PROBLEM

Let us replace γ in Eqs (2.1) by $\mu\gamma$ and let μ go to zero. This yields a system

$$\dot{\mathbf{M}}=\mathbf{M}\times A\mathbf{M},\quad \dot{\boldsymbol{\gamma}}=\boldsymbol{\gamma}\times B\mathbf{M} \tag{3.1}$$

describing the rotation of the body when the vortex strength is small compared with the angular momentum. The first vector equation of the system is independently integrable (the Euler–Poinsot problem). As to the second, after the known function $\mathbf{M}(t)$ has been substituted into it

$$\dot{\boldsymbol{\gamma}}=\boldsymbol{\gamma}\times B\mathbf{M}(t) \tag{3.2}$$

it becomes a linear Hamiltonian system on SO(3) with periodic coefficients. For it to be integrable we need to know one more first integral (apart from the geometrical one), which is periodic in t. Such an integral indeed exists, since the system is linear and Hamiltonian; but it is not known how to extend it to the general system (3.1), whose behaviour is nevertheless regular.

The additional integral (3.2) is multi-valued in the complex sense, that is, it is not holomorphic in $\mathbf{C}^3 \times X$, where X is the Riemann surface of the first three equations of (3.1). However, if the first condition of (2.7), obtained from the condition that the KIs of system are integers, holds for odd values of $k = k_a$, an additional first integral of system (3.2) may be found explicitly. It also then proves possible to find an explicit expression for the additional first integral of the general system (3.1).

We note that for system (3.1) to be integrable we still lack one more integral, besides the trivial integrals $I_1 = (\mathbf{M}, A\mathbf{M})$, $I_2 = (\mathbf{M}, \mathbf{M})$, $I_3 = (\boldsymbol{\gamma}, \boldsymbol{\gamma})/2$ and the standard invariant measure. It turns out that the construction is simpler after one defines additional fields of symmetries for system (3.1).

Let us introduce the differential operator defined by system (3.1)

$$\hat{D} = \mathbf{M} \times A\mathbf{M} \frac{\partial}{\partial \mathbf{M}} + \boldsymbol{\gamma} \times B\mathbf{M} \frac{\partial}{\partial \boldsymbol{\gamma}} \tag{3.3}$$

Then a field of symmetries will be defined by some operator $\hat{P}$ that commutes with $\hat{D}$ in the usual sense

$$[\hat{D}, \hat{P}] = \hat{D}\hat{P} - \hat{P}\hat{D} = 0 \tag{3.4}$$

We will seek $\hat{P}$ in the form

$$\hat{P} = \mathbf{P}(\mathbf{M})\partial / \partial\boldsymbol{\gamma} \tag{3.5}$$

Writing out condition (3.4), we obtain three linear partial differential equations defining a vector $\mathbf{P} = (P_1(\mathbf{M}), P_2(\mathbf{M}), P_3(\mathbf{M}))$

$$\hat{D}\mathbf{P} = \begin{Vmatrix} 0 & M_3 b_3 & -M_2 b_2 \\ -M_3 b_3 & 0 & M_1 b_1 \\ M_2 b_2 & -M_1 b_1 & 0 \end{Vmatrix} \mathbf{P} \tag{3.6}$$

The linear substitution

$$\mathbf{P} = \operatorname{diag}(M_1, M_2, M_3) A_1^{-1} K A_3^{-1} K \ldots A_{k-2}^{-1} K \mathbf{T} \tag{3.7}$$

reduces (3.6) to the form

$$\hat{D}\mathbf{T} = K A_k \mathbf{T} \tag{3.8}$$

where

$$A_n = \begin{Vmatrix} -a_{32} n & b_3 & -b_2 \\ -b_3 & -a_{13} n & b_1 \\ b_2 & -b_1 & -a_{21} n \end{Vmatrix}, \quad n = 1, 3, 5, \ldots \tag{3.9}$$

are constant matrices, and moreover

$$\det A_n = -n(n^2 a_{32} a_{13} a_{21} + b_1^2 a_{32} + b_2^2 a_{13} + b_3^2 a_{21}) \tag{3.10}$$

$k = \operatorname{diag}(M_1^2, M_2^2, M_3^2)$, where $k > 0$ is an odd number. Hence it follows that $\mathbf{T}$ is a constant vector, defined by the condition $A_k \mathbf{T} = 0$. Clearly, $\mathbf{T} \neq 0$ if $\det A_k = 0$, that is

$$k^2 a_{32} a_{13} a_{21} + b_1^2 a_{32} + b_2^2 a_{13} + b_3^2 a_{21} = 0 \tag{3.11}$$

(k is an odd number).

This is a sufficient condition for system (3.1) to have a non-trivial field of symmetries represented by the explicit expression (3.7).

The identity $\hat{D}I_3 \equiv 0$ immediately implies that also $\hat{D}(\hat{P}I_3) \equiv 0$, i.e. $\hat{P}I_3 = I_4$ is the required additional integral of Eqs (3.1) when condition (3.11) holds. We write this integral in the form

$$I_4 = (\mathbf{P}(\mathbf{M}), \boldsymbol{\gamma}) \tag{3.12}$$

It follows from the linearity of I_4 with respect to γ that it is independent of I_1, I_2 and I_3. In addition, it can be shown that, as a homogeneous form of order $k + 1$, I_4 cannot be reduced to a combination of integrals of lower degrees.

Condition (3.11) is satisfied in the case when $B = kA$, where k is an odd number. When $k = 1$ we obtain the Euler–Poisson equations, in which case $I_4 = (M, \gamma)$. When $k = 3$ we obtain, by (3.12)

$$I_4 = \Phi_1 + \Phi_2 + \Phi_3 \tag{3.13}$$

$$\Phi_1 = \gamma_1 M_1[(a_{13}a_{21} + 9a_1^2)M_1^2 + (3a_3a_{21} + 9a_1a_2)M_2^2 + (9a_1a_3 - 3a_2a_{13})M_3^2] \quad (1\,2\,3).$$

As pointed out before, the system of equations (3.1) has an additional fourth integral, whatever the elements of the matrices A and B. If (3.11) holds, it can be represented by a polynomial. It has also been possible to find an explicit expression in the case when $b_1 = b_2 = 0, b_3 \neq 0$, while condition (3.11) need not necessarily hold. Then

$$I_4 = \gamma_1 \sin\varphi + \gamma_2 \cos\varphi \tag{3.14}$$

$$\varphi = \frac{b_3}{\sqrt{a_{13}a_{32}}} \ln(\sqrt{a_{13}}\, M_1 + \sqrt{a_{13}}\, M_2)$$

This example once again confirms the fact that the general solution of Eqs (3.1), like their integrals, may be multi-valued.

Generally speaking, the following statement is true: the general solution of a system of equations (3.1) can be expressed in terms of single-valued functions of time if and only if condition (3.11) is satisfied.

It would be interesting to extend the above integrals of system (3.1) to system (2.1). We have not been able to do so, except in the case when $k = 1$, which is already known [12, 13]. It has been shown by numerical investigations using construction of the Poincaré mapping that such a generalization is impossible unless one imposes further restrictions on the elements of the matrices A, B and C.

Note that some examples of integrable systems in the real domain with branching solutions as functions of complex time may be found in [15].

4. GENERALIZATION OF CHAPLYGIN'S PROBLEM ON THE ROLLING OF A DYNAMICALLY ASYMMETRIC SPHERE

Consider the inertial motion without slipping of a Chaplygin sphere [16] on the surface of a sphere (Fig. 1). The equations of motion of such a system are

$$\dot{\mathbf{M}} = \mathbf{M} \times \boldsymbol{\omega}, \quad \dot{\boldsymbol{\gamma}} = \lambda\boldsymbol{\gamma} \times \boldsymbol{\omega}, \quad \lambda = R/(R-a) \tag{4.1}$$

$$(\mathbf{M} = I\boldsymbol{\omega} + D\boldsymbol{\gamma} \times (\boldsymbol{\omega} \times \boldsymbol{\gamma}))$$

where $\mathbf{M}$ is the angular momentum about the point of contact in a system of coordinates attached to the body, $\boldsymbol{\gamma}$ is the vector joining the centres of the two spheres, and I is the inertia tensor relative to the centre of mass. Equations (4.1) have first integrals $(\mathbf{M}, \mathbf{M}) = \text{const}$, $(\mathbf{M}, \boldsymbol{\omega}) = \text{const}$, $(\boldsymbol{\gamma}, \boldsymbol{\gamma}) = 1$ and an invariant measure found by Chaplygin

$$\mu = [1 - D(I^{-1}\boldsymbol{\gamma}, \boldsymbol{\gamma})]^{-1/2}$$

For these equations to be integrable, we still need one more first integral—the analogue of the area integral. Note that if $D = 0$, Eqs (4.1) reduce to system (3.1) with $B = \lambda A$, $A = I^{-1}$. We can therefore try to generalize the integral (3.12) to Eqs (4.1) when $D \neq 0$.

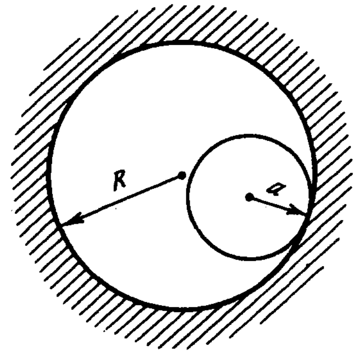

Fig. 1.

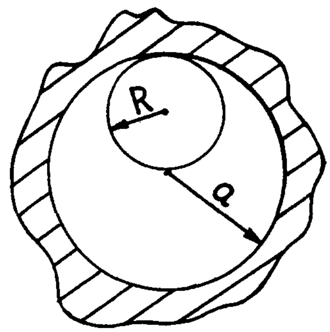

Fig. 2.

If $\lambda = 1$ ($R \Rightarrow \infty$), we obtain the classical Chaplygin problem, which is integrable when $\lambda = -1$ ($a = 2R$) corresponding to a fixed sphere, with a dynamically asymmetric sphere of twice the sphere's radius rolling around it on the outside (Fig. 2).

It turns out that integral (3.12) with $B = -A$ also holds for system (4.1). Hence the problem is integrable. However, it is not an essentially new integrable problem of non-holonomic mechanics, since its equations may be reduced, by a linear change of variables, to the form of Eqs (4.1) with $\lambda = 1$ [17]. When $\lambda = 1$ one also obtains an integrable generalization of system (4.1) if one introduces the force field of the Bruns problem, but it is difficult to suggest a physical interpretation for that field in this case.

We were unable to find any generalizations of the integral (3.12) to system (4.1). It later turned out that this cannot be done directly, because, generally speaking, the behaviour of system (4.1) when $\lambda = 2n + 1$ ($n \in \mathbb{Z}$) is stochastic. This has been confirmed by numerical studies of the Poincaré mapping.

We have also been unable to extend the area integral to the case of a balanced gyrostat, even for $\lambda = -1$.

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